Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name : Engineering Mathematics-II
Subject Code : 4TE02EMT2 **Branch: B.Tech (All)**
Semester : 2 **Date :04/05/2017** **Time : 02:00 To 05:00** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q.1 Attempt the following questions: **(14)**

a) $\int_0^{\frac{\pi}{2}} cos^7\theta\, d\theta$ = _______.

(a) 0 (b) $\frac{16}{35}$ (c) $\frac{32}{35}$ (d) $\frac{8\pi}{35}$.

b) If $f_n = \int_0^{\frac{\pi}{4}} \tan^n \theta\, d\theta$ then $f_n + f_{n-2}$ = _______.

(a) $\frac{1}{n}$ (b) $\frac{1}{n-1}$ (c) $\frac{n}{n-1}$ (d) $\frac{n-1}{n}$.

c) State the Euler's formula.

d) $\Gamma(4.5)$ = _______.

(a) 4.5 $\Gamma(3.5)$ (b) 3.5 $\Gamma(4)$ (c) 4 $\Gamma(3.5)$ (d) none of these.

e) $\beta\left(\frac{1}{2},\frac{1}{2}\right)$ = _______.

(a) $\sqrt{\pi}$ (b) $\frac{1}{2}$ (c) 1 (d) π.

f) Define a complete elliptical integral of 1^{st} kind.

g) $erf(x) + erf_c(x)$ = _______.

(a) 0 (b) 1 (c) 2 (d) none of these.

h) The curve $y^2 (2a – x) = x^3$ is symmetrical about _______.

(a) X-axis (b) Y-axis (c) origin (d) line y = x.

i) The length of the spiral $r = e^{\theta}/\sqrt{2}$, $0 \le \theta \le \pi$, is _______.

(a) e^{π} (b) $e^{\pi} + 1$ (c) $e^{\pi} – 1$ (d) 1.

j) $\int_0^{\pi}\int_0^{x} x \sin y\, dy\, dx$ = _______.

(a) $\pi/2$ (b) $\pi^2/2$ (c) $(\pi/2) + 1$ (d) $(\pi^2/2) + 1$.

k) $\int_0^1 \int_0^1 \int_0^1 (x^2 + y^2 + z^2)\, dzdydx$ = ________.

(a) 1 (b) -1 (c) 0 (d) none of these.

l) The series $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}$ is ________.

(a) convergent (b) divergent (c) conditionally convergent

(d) none of these.

m) The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is divergent if ________.

(a) $p > 1$ (b) $p \le 1$ (c) $p \ge 1$ (d) $p < 1$.

n) The differential equation $(x + y)\, dy + (x - y)\, dx = 0$ is ________ differential equation.

Attempt any four questions from Q-2 to Q-8

Q.2 Attempt all questions **(14)**

a) Evaluate: **(02)**

$\int_0^{\pi} (1 + \cos\theta)^4\, d\theta$.

b) Evaluate: **(02)**

$\int_0^{\frac{1}{2}} x^3\sqrt{1 - 4x^2}\, dx$.

c) Evaluate: **(04)**

$\int_0^1 \frac{x^6}{(1 + x^2)}\, dx$.

d) Trace the curve $x^3 + y^3 = 3axy$. **(06)**

Q.3 Attempt all questions **(14)**

a) If $\beta(n, 3) = 1/105$ and n is a positive integer, then find n. **(02)**

b) Prove that: (i) $n\beta(m + 1, n) = m\beta(m, n + 1)$; and **(04)**

(ii) $\beta(m, n) = \beta(m, n + 1) + \beta(m + 1, n)$.

c) Evaluate: **(04)**

$\int_{-\infty}^{\infty} e^{-k^2x^2}\, dx$.

d) Prove that: **(04)**

$\int_0^{\infty} \frac{x^4}{4^x}\, dx = \frac{24}{(\log 4)^5}$.

Q.4 Attempt all questions **(14)**

a) Prove that: **(03)**

$$\int_0^{\infty} \frac{\sqrt{x}}{x^2 + 2x + 1}\, dx = \frac{\pi}{2}.$$

b) Evaluate: **(03)**

$$\int_0^{\frac{\pi}{3}} \frac{d\theta}{\sqrt{3 - 4\sin^2\theta}}.$$

c) If the perimeter of the ellipse of e = 1/√2 is equal to twice the length of one arch of the curve y = sin x, then find the area of the ellipse. **(04)**

d) (1) Show that erf(x) is an odd function; and **(04)**

(2) prove that:

$$\int_{-a}^{a} e^{-t^2}\, dt = \sqrt{\pi}\,\mathrm{erf}\,(a).$$

Q.5 Attempt all questions **(14)**

a) Trace the curve $r^2 = a^2 \cos 2\theta$. **(04)**

b) Find the length of the Cardioid $r = 1 + \cos\theta$. **(02)**

c) Find the area of the smaller region lying above X-axis and bounded by the circle $x^2 + y^2 = 2x$ and the parabola $y^2 = x$. **(04)**

d) Find volume of the solid generated by revolving the lemniscate $r^2 = a^2 \cos 2\theta$ about the line $\theta = \pi/2$. **(04)**

Q.6 Attempt all questions **(14)**

a) Evaluate: **(02)**

$$\iint (x^2 + y^2)\, dA$$

, where R is a triangular region with vertices (0, 0), (0, 1) and (1, 0).

b) Evaluate: **(03)**

$$\int_0^1 \int_y^1 x^2 e^{xy}\, dxdy.$$

c) Evaluate: **(03)**

$$\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln(x^2 + y^2 + 1)\, dxdy.$$

d) Find the volume of the region D between the cylinder $z = y^2$, and the XY-plane that is bounded by the planes x = 0, x = 1, y = -1, y = 1. **(03)**

e) Evaluate: **(03)**

$$\int_0^1 \int_{\sqrt[3]{z}}^{1} \int_0^{\ln 3} \frac{\pi e^{2x} \sin(\pi y^2)}{y^2}\, dxdydz.$$

Q.7 Attempt all questions **(14)**

a) Discuss convergence/divergence of the following series: **(08)**

$i)\ \sum_{n=1}^{\infty}\frac{2n+1}{n^2(n+1)^2}$ $ii)\ \sum_{n=1}^{\infty}\frac{1}{2\sqrt{n}+\sqrt[3]{n}}$ $iii)\ \sum_{n=1}^{\infty}\frac{n^{10}}{10^n}$ $iv)\ \sum_{n=1}^{\infty}(-1)^n\ln\left(1+\frac{1}{n}\right)$

b) Prove that if the series Σa_n converges, then **(02)**

$\lim_{n\to\infty} a_n = 0$

c) Find the values of x for which the following power series converges. **(04)**

$$\sum_{n=1}^{\infty}(-1)^n\frac{(x+2)^n}{n}.$$

Q.8 Solve the following ordinary differential equations: **(14)**

a) $(x - y)dx - (x + y)dy = 0$ **(03)**

b) $(y - x^3)dx + (x + y^3)dy = 0$ **(03)**

c) $(1 + x^2)dy + 2xy\ dx = \cot x\ dx$ **(04)**

d) $2yy'' = 1 + (y')^2$ **(04)**